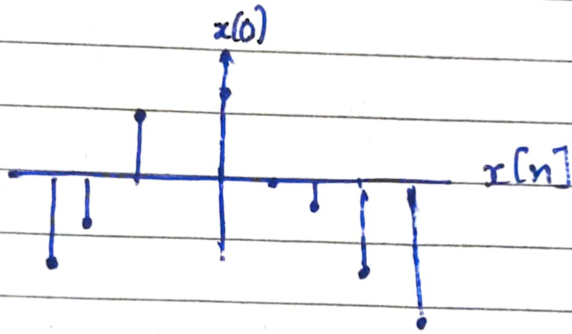
EE-201 Signals and Systems

Marking Scheme:

6 Quizzes → 30 Marks
Midsem → 30 marks
Endsem → 40 marks

Continuous and discrete signals:

1) Continuous Time (CT signals) $x[t]$
2) Discrete Time (D T signals) $x[n]$



Complex number: $z\angle\theta = re^{i\theta}$

$= x+iy$

here $r = \sqrt{x^2+y^2}$

if θ is in degrees convert it to radians in the final answer. (in terms of π)

$$e^{j\phi} = \cos\phi + \sin\phi\, j$$

Rectangular form: $a+jb$

polar form → $r\angle\theta$, $re^{j\theta}$

$-\pi < \theta < \pi$

Signal energy and power:-

Energy = Power × Time

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\,dt \times (t_2 - t_1)$$

$$\Rightarrow \int_{t_1}^{t_2} p(t)\,dt$$

$$\text{power} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\,dt = \int_{t_1}^{t_2} v^2(t)\,dt$$

=

for C.T. the total energy over time interval $t_1 \le t \le t_2$ can be given by:

$$E = \int_{t_1}^{t_2} |x(t)|^2\,dt$$

for D.T. it will be over the time interval $n_1 \le n \le n_2$

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2$$

$$P_{avg.} = \frac{1}{T} \int_{0}^{T} p(t)\,dt$$

Q $x[n] = \cos\left(\frac{\pi}{4} n\right)$

$$E_\infty = \sum_{n=1}^{\infty} \cos\left(\frac{\pi n}{4}\right)^2 = 1 + 1 + 1 + \cdots$$

$$= \infty .$$

Signals and Systems.

Signal
- Continuous time
- disc. time

Signals
- deterministic → can be predicted accurately
- Random → • comm. system • Data gen. sys. • Noise.

Energy = $\int_{t_1}^{t_2} |x(t)|^2 dt$

$E_\infty < \infty$ → finite energy (Energy signal)

eg. Sine integral → if considered b/w two fixed lims

$0 <$ Power $< \infty$ {Power signal}

Periodic Signal → If $(x(t+T) = x(t))$, it is called a periodic signal and T is the period of the signal

Signal Energy and Power :→

Energy → $\int_{t_1}^{t_2} p(t)\,dt$

Power → $\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\,dt$

for continuous signal, over time interval $t_1 < t < t_2$ can be given by

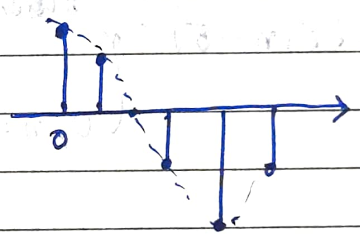
$$E = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

for discrete it will be over the time interval $n_1 \leq n \leq n_2$.

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2 \qquad P_{avg.} = \frac{1}{T} \int_0^T p(t) \, dt$$

$$P_\infty = \frac{1}{\quad}$$

Stem plot :→



Any signal can be written as sum of an even & odd signal.

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

Unit delta f^n :-

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

Unit step funct. :-

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$u[n] = \sum_{k=-\infty}^{\infty} \delta[k] \qquad \delta[n] = u[n] - u[n-1]$$

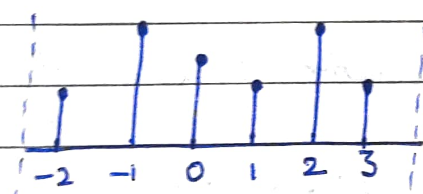
$$x[n]\,\delta[n] = \begin{cases} x[n], & n = 0 \\ 0, & n \neq 0 \end{cases}$$

⇒ used to extract value of f^n at $n=0$

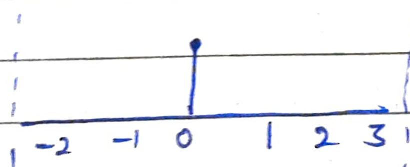
$$x[n]\,\delta[n-5] = \begin{cases} x[5], & n = 5 \\ 0, & n = 0 \end{cases}$$

⇒ used to extract value of f^n at $n=5$

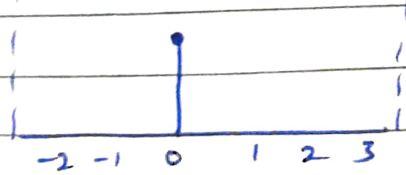
ex. $x[n]$ =



$\delta[n]$

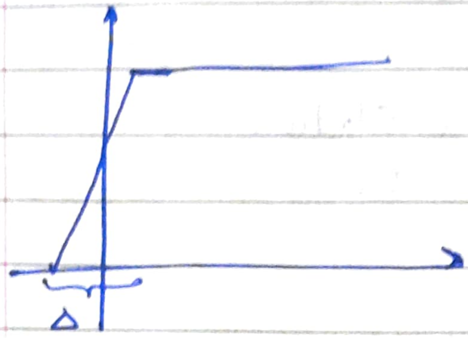


$x[n]\,\delta[n]$



To multiply two signals, we take product of two signals at all times t.

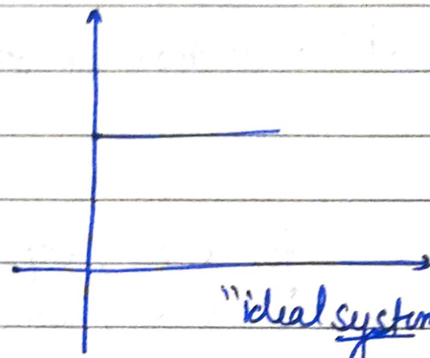
Unit step in all physical systems is cont.



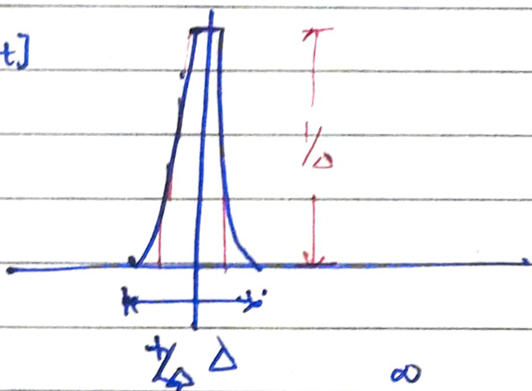
$U_\Delta(t) \rightarrow$ transition from $0 \rightarrow 1$.

for ideal systems

$\Delta \rightarrow 0$.



$\delta_\Delta[t]$



$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

The amplitude of this impulse $\rightarrow \infty$.

But; $\forall \Delta$; $\int_{-\infty}^{\infty} \delta(t)\,dt \equiv 1$; unit impulse

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau. \Rightarrow \delta(t) = \frac{d}{dt} u[t]$$

Complex Representation:→

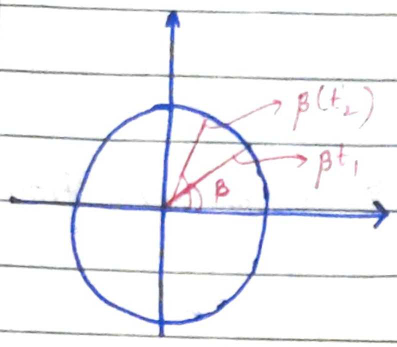
$x(t) = A e^{(\sigma + \beta j)}$

$A \rightarrow$ complex amp.

$y(t) = e^{j\beta t}$

$= \cos(\beta t) + j \sin(\beta t)$

β: angular velocity



move in anticlockwise dir.

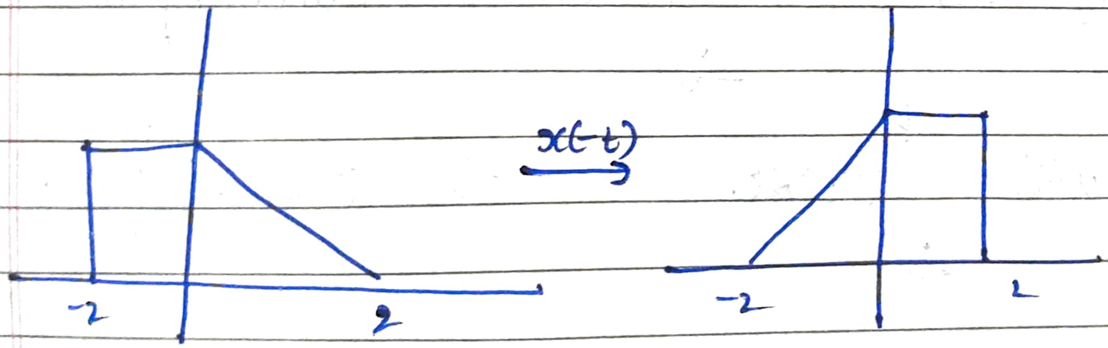
$e^{-j\beta t} \longrightarrow$ move in other direction.

$e^{\sigma t}$ {real signal}
for $\sigma > 0$; expanding
$\sigma < 0$; converging.

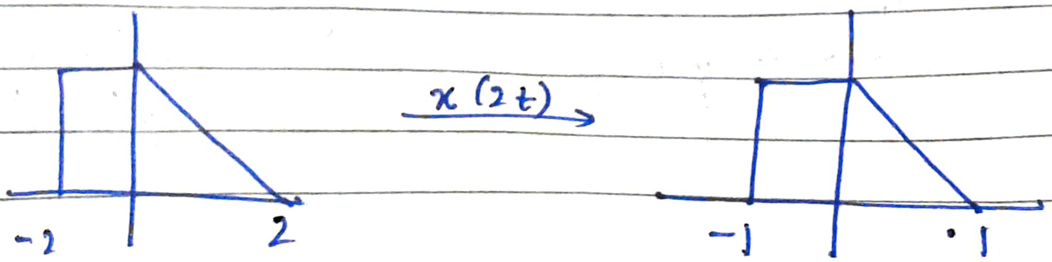
for a complex equation, we get a damped oscillation.

SIGNAL TRNSF :-

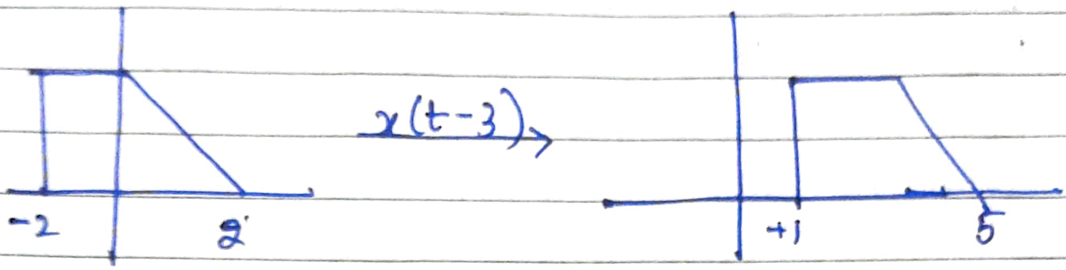
(i) Time inversion



(ii) scaling



(ii) Shifting



- All operations are done on t.

for discrete signals shift must be integer, because discrete signals are valid for integers only.

→ Quantization of signal.

Q. Find energy and power of following signal:-

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

⇒ $u[n] = 0 \; ; \; n<0$

$\quad 1 \; ; \; n>0$

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n ; & n>0 \\ 0, & n<0 \end{cases}$$

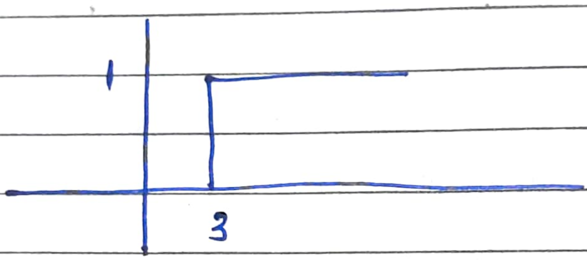
~~for~~ energy $= (1)^2 + \left(\frac{1}{2}\right)^2 + \dots + \infty$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

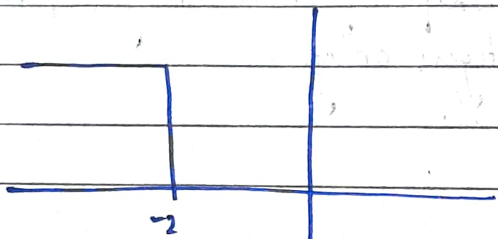
power $= \frac{4}{3N}$ as $N \to \infty$, power $\to 0$.

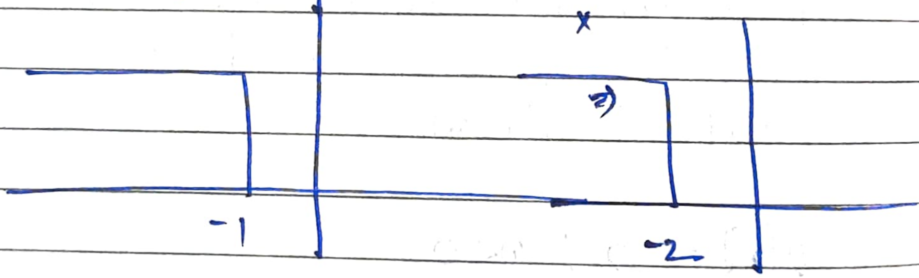
Q Given $x(t)$, find $x(1-t)$ $x(2-t)$

$x(t) \longrightarrow$



$x(1-t)$





Q find periodicity of $x[n]$

$$x[n] = \sum_{k=-\infty}^{\infty} \{ \delta[n-4k] - \delta[n-1-4k] \}$$

$$= \sum_{k=-\infty}^{\infty} \{ \delta[n+N-4k] - \delta[n+N-1-4k] \}$$

$$= \sum_{k=-\infty}^{\infty} \{ \delta[n+4(\frac{N}{4}-\frac{k}{4})] - \delta[n-1+4(\frac{N}{4}-k)] \}$$

$n \in 4k$

Signals → vary with time and space
↳ Interacting with a physical system.

$$\text{I/P} \longrightarrow \boxed{\text{System}} \xrightarrow{\text{O/P}}$$

Signal

- continuous → continuous time system
- discrete → discrete time system

$T()$ → maps input to output

$T() = x^2$

$y(t) = T(x(t))$ $\qquad x[n] \xrightarrow{T()} y[n].$

ways of connecting two systems :-

1) Series: (Cascade)

$$x(t) \longrightarrow \underset{T_1(x(t))}{\boxed{T_1()}} \longrightarrow \underset{T_2(T_1(x(t)))}{\boxed{T_2()}} \longrightarrow \dots \longrightarrow y(t)$$

"On changing order response changes"

eg :-

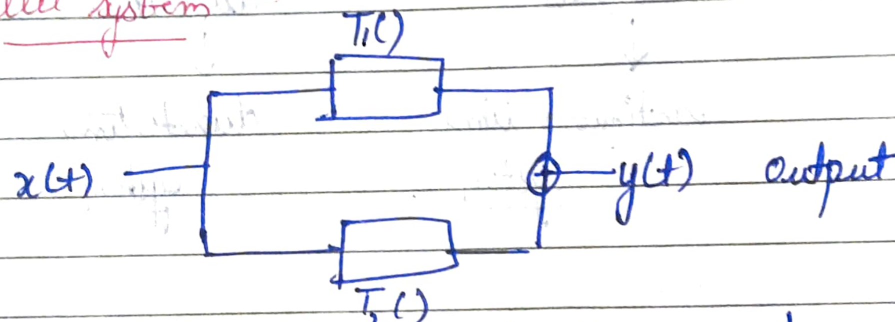
$$x(t) \longrightarrow \boxed{\sqrt{x(t)}} \longrightarrow \boxed{2x(t)} \longrightarrow 2\sqrt{x(t)}$$

On interchanging

$$x(t) \longrightarrow \boxed{2x(t)} \longrightarrow \boxed{\sqrt{x(t)}} \longrightarrow \sqrt{2x(t)}$$

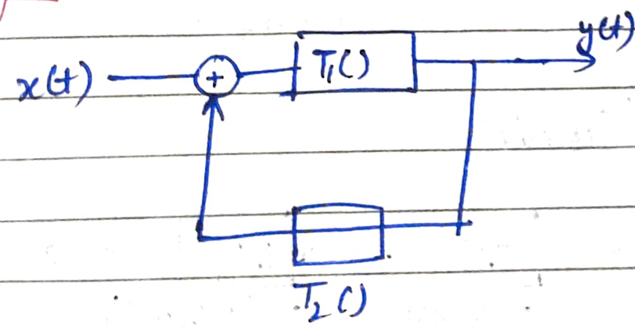
Hence on changing the order of system; respons changes

→ Parallel System :→



The response doesn't depend on order of system

→ Feedback System :→



This is mostly used to correct errors.

→ It can be used to make an unstable system; stable.

Properties of various systems :→

(I) memory less or have memory :→

$y(t_1)$ depends on $x(t_1)$; like $y(t_1) = x(t_1)^2$

$$x(t) \rightarrow \boxed{T(\,)} \xrightarrow[y(t)]{} = \int_{\infty}^{t} x(\tau)\,d\tau \Rightarrow \text{has a memory}$$

↳ depends on all the past values

(II) Invertibility :→

$$\begin{tikzcd} x(t) \arrow[r] & \boxed{T_1(\,)} \arrow[r] & y(t) \arrow[d] \\ & & \boxed{T_2(\,)} \arrow[llu] \end{tikzcd}$$

This system is invertible if we can find another system ~~st~~ such that when it acts on $y(t)$, it gives $x(t)$ back.

eg1 $T_1(\,) = \int_{-\infty}^{t} x(\tau)\,d\tau$ is invertible by the f^n $\frac{d}{dt}x(t)$

eg.2 $T_1(\,) = x_1^2$ is not invertible b/cs $\sqrt{x_1(t)}$ gives +ve values only.

eg.3 $T_1(\,) = 2x(t) + 3$ is invertible because of the f^n $\frac{x(t)-3}{2}$.

(III) Causality :→

Consider a discrete time system :→

$$y[n] = \frac{x[n-1] + x[n] + x[n+1]}{3}$$

o/p depends on the signal value in the future.
↳ Non-causal.

$T() = x(-t) \Rightarrow$ Non causal.

$t(-2) = x(2)$ ✗

$t(2) = x(-2)$ ✓

(iv) Stability

BIBO → Bounded input gives bounded output.

$T() = x^2(t)$

system is stable for a bounded input.

(v) Linearity :→ → super position
→ Homogenity.

$x_1[n] \longrightarrow y_1[n]$

$x_2[n] \longrightarrow y_2[n]$

$x_1[n] + x_2[n] \longrightarrow y_1[n] + y_2[n]$

$\cancel{x_1}$ $T() = 2x(t) + 3$

Homogeniety:→

$x(t) \longrightarrow y(t)$

$\alpha x(t) \longrightarrow \alpha y(t)$

↓

can be complex.

Time invariance :-

$$x(t) \rightarrow y(t)$$
$$x(t-t_0) \rightarrow y(t-t_0) \qquad \{\text{Time invariant}\}$$

eg. $T() = \sum_{k=-\infty}^{n} x[k]$ (Accumulative)

$$y[n-t_0] = \sum_{k=-\infty}^{n} x[k-t_0]$$

if the above is not true it is time variant.

eg. $\boxed{T() = t\,x(t)}$

→ $y(t) = \frac{d}{dt}x(t)$ is not an invertible system

$\int_{-\infty}^{t} y(t)\,dt = x(t) + c$, where c is an unknown.

Q $y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$

- Has memory
- $y\left(\frac{1}{2}\right) = \int_{-\infty}^{1} x(\tau)\,d\tau \rightarrow$ non causal
- If ~~x)~~ $x(t) \leq K$

$$\int_{-\infty}^{2t} x(\tau)\,d\tau \leq K\int_{-\infty}^{2t} \cancel{K}\,d\tau \Rightarrow \text{not stable}$$

(N) It is a linear sy.

(V) $y(t-t_0) = \int_{-\infty}^{2(t-t_0)} x(\tau)\, d\tau$ { assuming invariance }

$= \int_{-\infty}^{2t} x(t-t_0)\, d\tau$

$t - t_0 = k$

$t = k + t_0$

$= \int_{-\infty}^{2(k+t_0)} x(k)\, d\tau$

Hence it is time variant.

- Has memory
- unstable.
- Non-causal
- Additive.
- Homogeneous
- Time Varying

Q. $y(t) = \sin(x(t))$

- memory less
- for $x(t) \leq k$; $-1 < \sin(x(t)) < 1$; stable
- causal system
- Additivity

$$x_1(t_1) \longrightarrow \sin(x_1(t))$$
$$x_2(t_2) \longrightarrow \sin(x_2(t))$$
$$x_1(t) + x_2(t) \longrightarrow \sin(x_1(t) + x_2(t)) \neq \sin(x_1(t)) + \sin(x_2(t))$$

- Homogenicity

$$x_1(t) \longrightarrow \sin(x_1(t))$$
$$\alpha x_1(t) \longrightarrow \sin(\alpha x_1(t)) \neq \alpha \sin(x_1(t))$$

Hence non linear.

- $$y(t) = \sin(x(t))$$
$$y(t-t_0) = \sin(x(t-t_0)) = y_1(t-t_0)$$

Hence time invariant

- Invertible:→ $(-\pi, \pi)$ {principle domain}
  If $x(t) \in$ ~~(-1,1)~~ , invertible

LTI systems:→
Linear Time Invariant system {additivity, homogenicity, Time invariant}

Q. $y(t) = \text{odd}\{x(t)\} = \dfrac{x(t) - x(-t)}{2}$

→ $x(2) = \dfrac{x(2) - x(-2)}{2}$; has memory

→ The function is stable

→ non causal system

$$y(t-t_0) = \frac{x(t-t_0) - (x(-(t-t_0)))}{2}$$

$$= \frac{x(t-t_0) - x(-t+t_0)}{2}$$

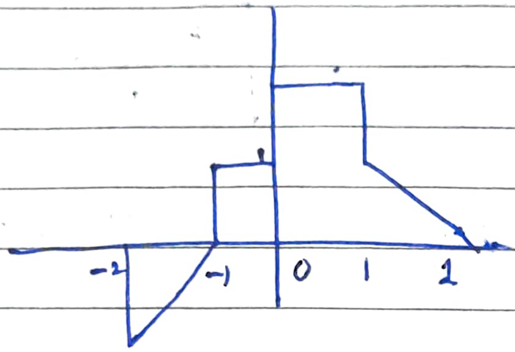
$$x_1(t) = x(t-t_0)$$

$$\Rightarrow \cancel{\frac{x(t-t_0) - x(-t \mp t_0)}{2}} \quad \frac{x(t-t_0) - x(t_0-t)}{2}$$

Hence it is time ~~invariant~~ variant.

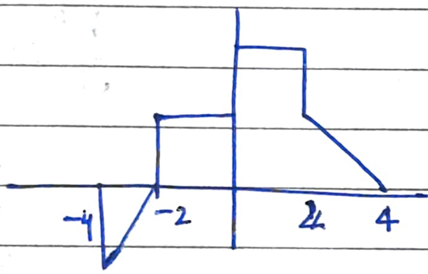
Tutorial-1

(1) $x(t) \rightarrow$



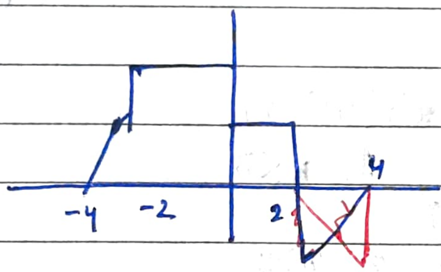
$\downarrow$

$x\left(\frac{t}{2}\right) \rightarrow$



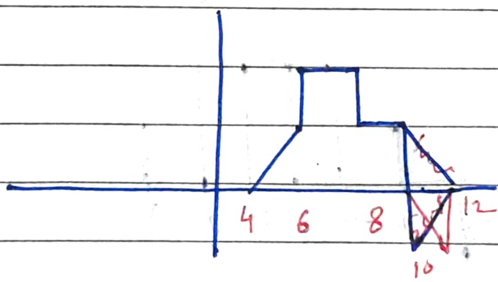
$\downarrow$

$x\left(-\frac{t}{2}\right) \rightarrow$

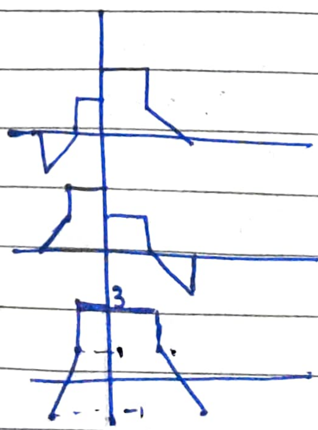


$x\left(-\frac{(t-8)}{2}\right) \rightarrow$

$x\left(4-\frac{t}{2}\right) \Rightarrow$

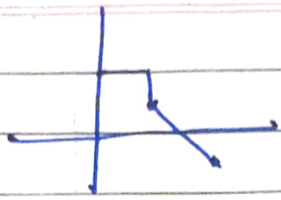


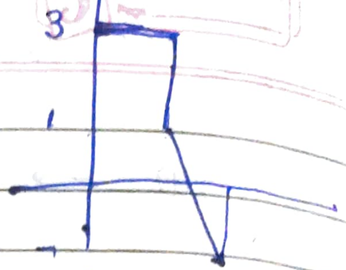
(b) $\{x(t)+x(-t)\}\,u(t)$



line + line → line

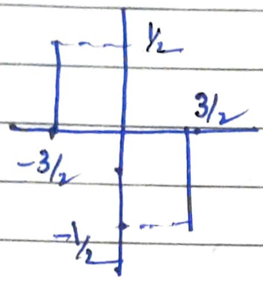
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$$





c) $x(t) \left\{ \delta\left(t+\frac{3}{2}\right) \cancel{+} \delta\left(t-\frac{3}{2}\right) \right\}$

$$\delta(t+a) = \begin{cases} 1, & t = -a \\ 0, & \text{otherwise} \end{cases}$$



$f_R(l) = x+y = 2 \quad \{x \in (1,2)\}$

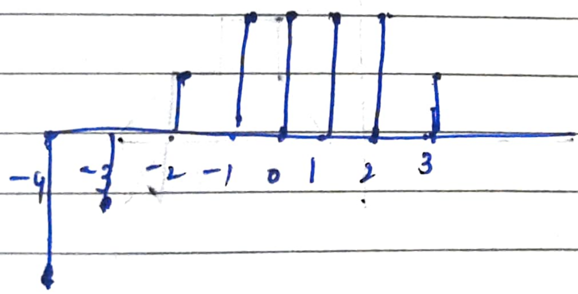
$f_L(l) = \cancel{2}x - y = -1$

$y_{3/2} = \frac{1}{2}$

$y_{-3/2} = -\frac{1}{2}$

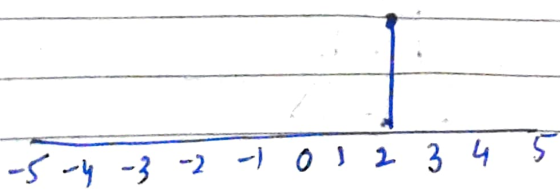
~~Q3~~ a) ~~f(n)~~ $x[n] = u[n]\,u[3-n]$

$$u[3-n] = \begin{cases} 1, & n \leq 3 \\ 0, & n > 3 \end{cases}$$



(b) $x[n] = x[n-2]\,\delta[n-2]$

$$\delta[n-2] = \begin{cases} 1, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

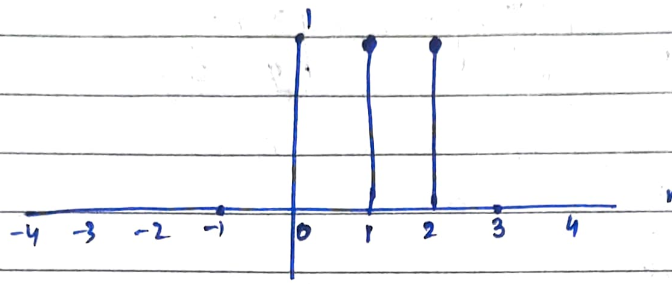


c) $x[(n-1)^2]$ $\Rightarrow$ $x[0] = x[1]$ $\quad$ $x[-1] = x[4]$

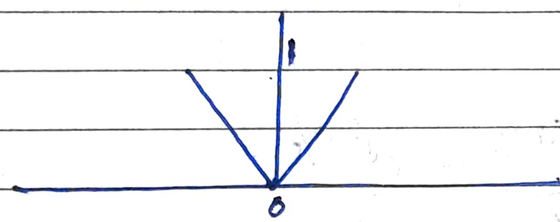
$x[1] = x[0]$ $\quad$ $x[-2] = x[9]$

$x[2] = x[1]$

$x[3] = x[4]$



(3) for even part of $f^n \rightarrow$ $f(x) = f(-x)$



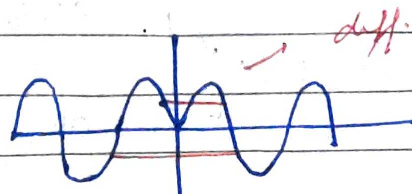
There is no odd part in the f^n.

(4) $\cancel{\cos(2\pi}$ $\cos\left(2t - \frac{\pi}{3}\right)^2$

(b) $\text{Even}\{\sin(4\pi t)u(t)\} = \dfrac{\sin(4\pi t)u(t) + \sin(-4\pi t)u(-t)}{2} = x(t)$

$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$ $\qquad$ $u(t) = \begin{cases} 1, & t < 0 \\ 0, & t \geq 0 \end{cases}$

$x(t) = \begin{cases} \dfrac{\sin(4\pi t)}{2} \\ \dfrac{-\sin 4\pi t}{2} \end{cases}$



diff. fundamental period

$f(N * \text{fundamental}) \equiv$ same values

d) $\cos\left[\frac{\pi}{8} n^2\right]$ is not periodic

e) $\underbrace{2\cos\left[\frac{\pi}{4} n\right]}_{P_1} + \underbrace{\sin\left[\frac{\pi}{8} n\right]}_{P_2} - \underbrace{2\cos\left[\frac{\pi}{2} n + \frac{\pi}{6}\right]}_{P_3}$

$\Rightarrow$ $P_1 = \frac{2\pi}{\pi/4} = 8.$

$P_2 = \frac{2\pi}{\pi/8} = 16$ Hence period $= 16$

$P_3 = \frac{2\pi}{\pi/2} = 4$

f) $x[n] = \underbrace{e^{j4\pi n/7}}_{P_1} - \underbrace{e^{j2\pi n/5}}_{P_2}$

$P_1 = \frac{2\pi}{4\pi/7} = \frac{7}{2}$

$P_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$

Period $= \text{LCM}\left\{\frac{7}{2}, \frac{5}{2}\right\} = \frac{35}{2}$

Hence the signal is periodic with period $\frac{35}{2}$

c) $\sum_{t=-\infty}^{\infty} e^{-(2t-t_0)} u(2t-t_0)$

$u(2t-t_0) = \begin{cases} 1, & t > t_0/2 \\ 0, & \text{other way} \end{cases}$

③

$\Rightarrow$

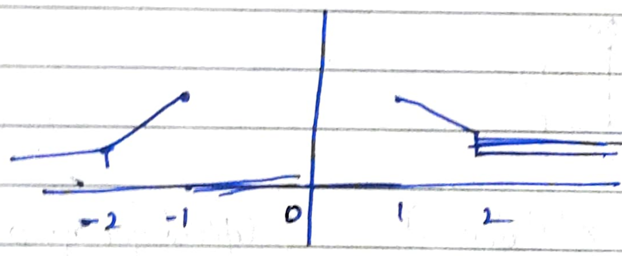
$$\Rightarrow \sum_{t=t_0/2}^{\infty} e^{-(2t-t_0)}$$

(3) $x[n] = \begin{cases} y-x=2, \ \cancel{yt} \ x\in(-2,-1) \\ x+y=0, \quad x\in(-1,0) \\ x=y \quad, x\in(0,1) \\ 1, \quad t>1 \end{cases}$

$$\text{even}\{x[n]\} = \frac{x(t)+x(-t)}{2}$$

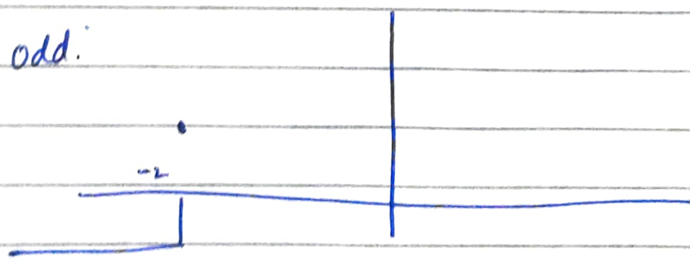
$$\text{for } x\in(-2,1) = \cancel{y-x}\ \frac{x+2+1}{2} = \frac{x+3}{2}$$

$$\text{for } x\in(-1,0) = \frac{\cancel{3}-x+x}{2} = 0$$



$$\frac{x+2-1}{2} = \frac{x+1}{2}$$

odd:



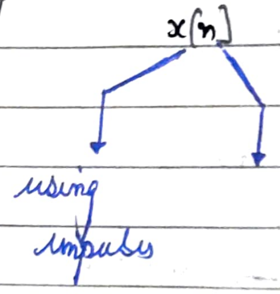
Interaction ~~and~~ of a Signal with a System :-

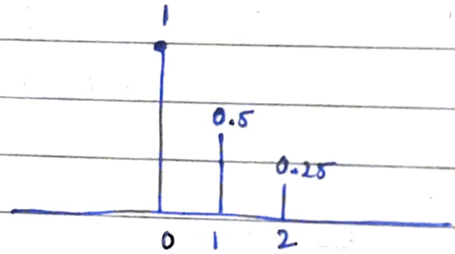
- Linear Time Invariant Systems

a) Linearity $\rightarrow$ additivity & homogenicity

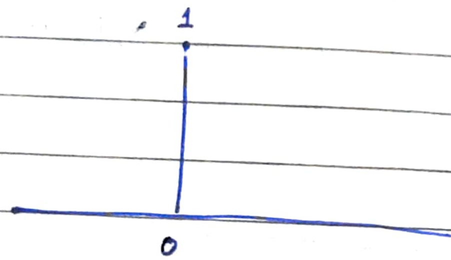
b) Time Invariance

1) Discrete signal :-

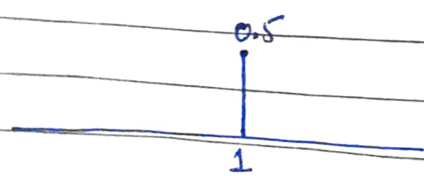




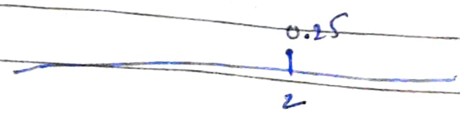
$x_0[n=0] = x[0]\,\delta[n-0]$ {defined for $n=0$}



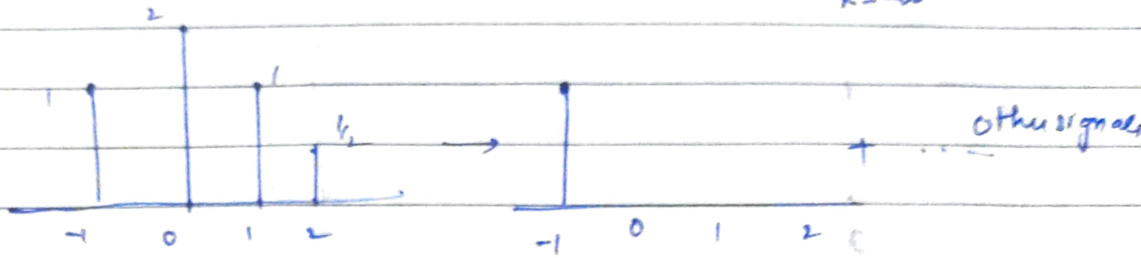
$x_1[n=1] = x[1] \cdot \delta[n-1]$



$x_2 = x[2]\,\delta[n-2]$



$\Rightarrow \quad x_k[n] = x[k]\,\delta[n-k] \quad \Rightarrow \quad x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$



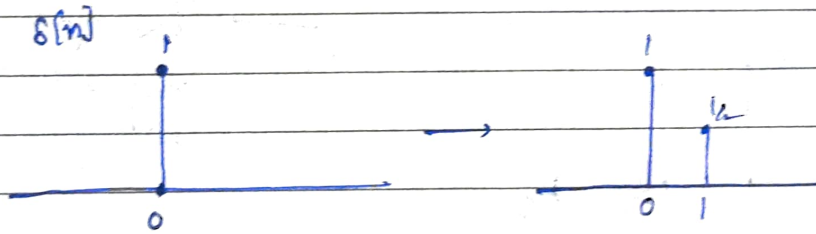
$\cancel{x[n] = x_0[\delta[\ x_0[\delta[n]] + x_{-1}}$

$x[n] = x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2]$

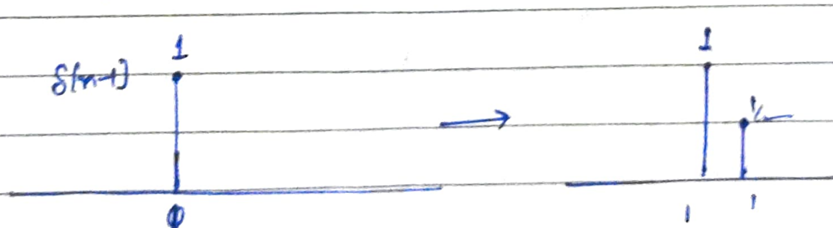
$x[n] \rightarrow \boxed{S} \rightarrow y[n]$

$h_0[n] =$

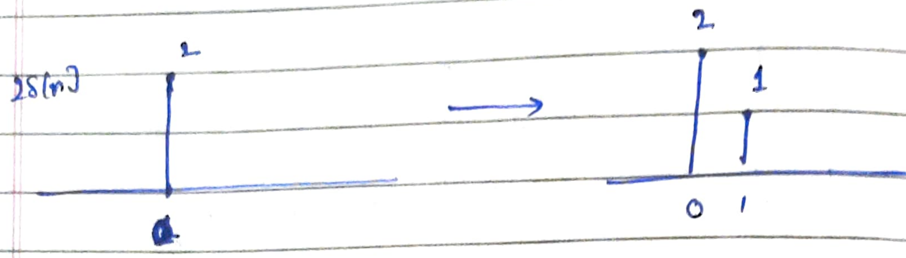
$\delta[n] \rightarrow \boxed{S} \rightarrow h_0[n]$



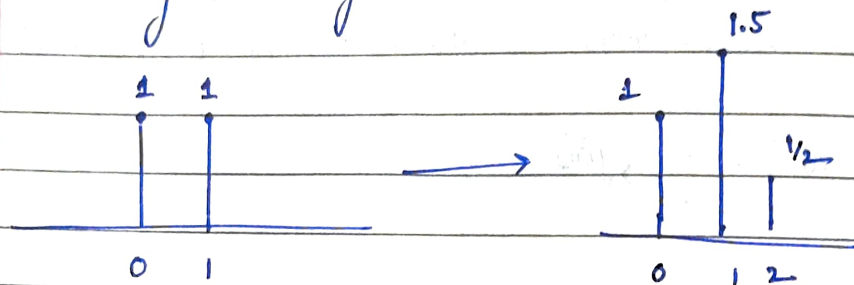
If system S is invariant



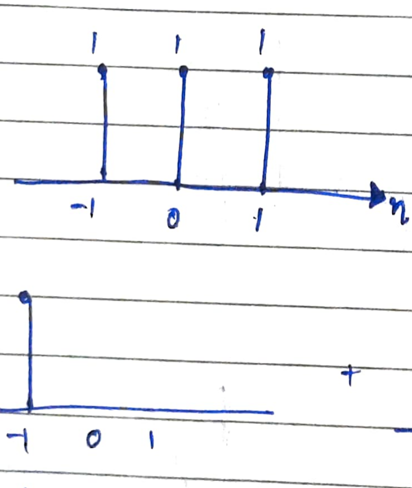
If we pass $2\delta[n]$ as the inputs



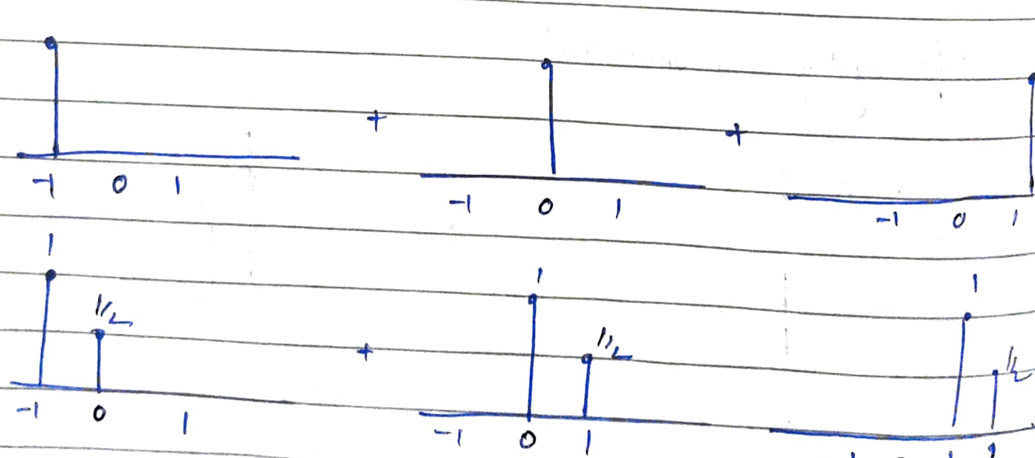
On adding the signals $\delta[n]$ & $\delta[n-1]$



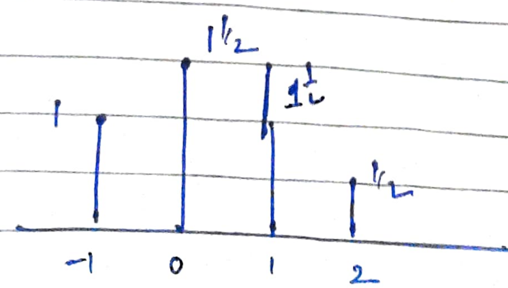
Q



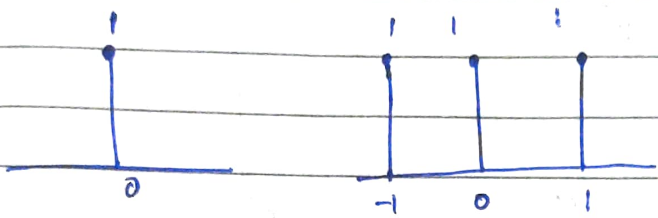
- break signal
- write individual output
- add

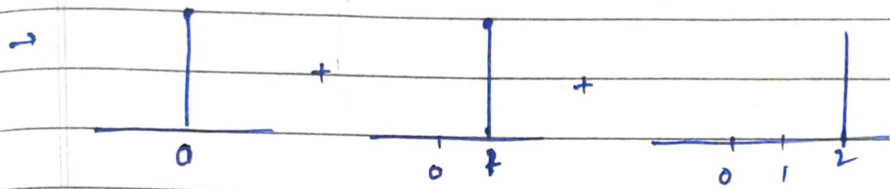


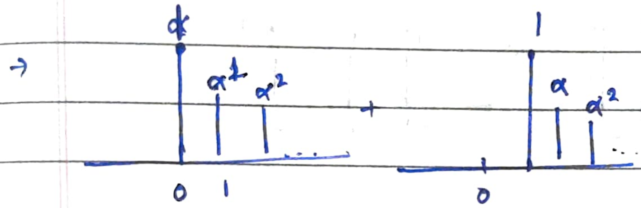
=>

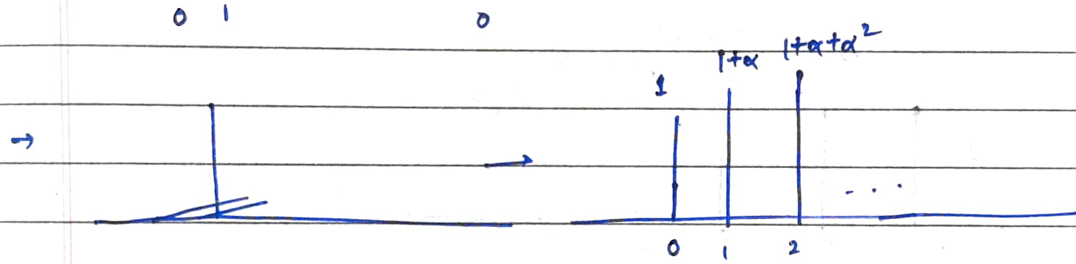



δ[n]
h₀[n]

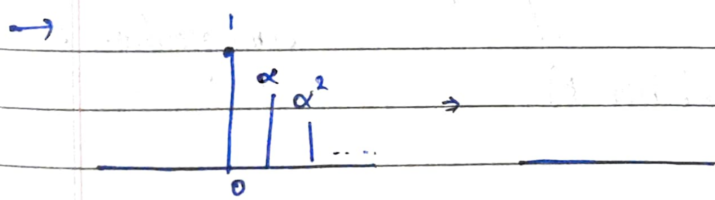

1
0
1 1 1
-1 0 1

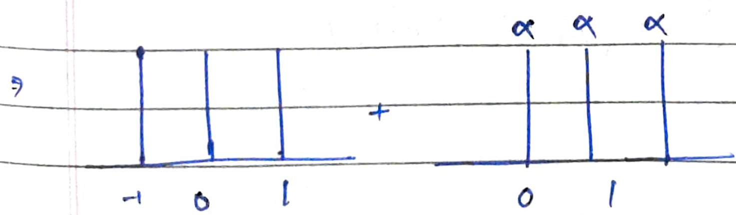

0
+
0 1
+
0 1 2


α
α¹ α²
0 1
+
1
α α²
0


1 1+α 1+α+α²
0 1 2

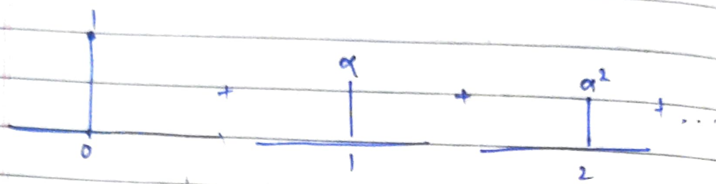
Σ


1
α α²
0

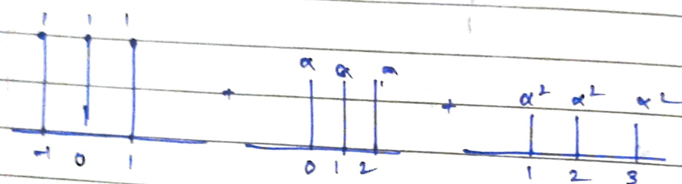

-1 0 1
+
α α α
0 1

Sol $x[n] = \alpha^n u[n]$

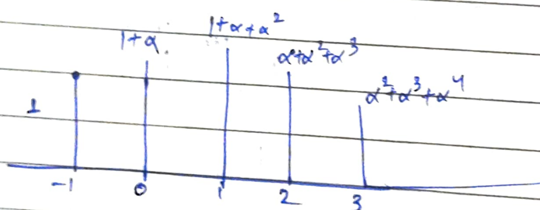
$$= \sum_{k=0}^{\infty} \alpha^k \delta[n-k]$$



→



=



$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$ → convolution sum or superposition sum

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] \left(u[n-k+1] - u[n-k-1] \right)$$

Tutorial - 3

S :

1 (a) $y(t) = x(t-2) + x(2-t)$

1. $y(3) = x(1) + x(-1)$, hence at a later duration of time it depends on past $\Rightarrow$ memory

2. $y(-1) = x(-3) + x(3)$; hence at $t = -1$ it depends at a future value $\Rightarrow$ non causal.

3. $y(t_1) + y(t_2) = x(t_1-2) + x(2-t_1) + x(t_2-2) + x(2-t_2)$
$= y(t_1) + y(t_2)$ {superimposable}

$y(\alpha x(t)) = \alpha x(t-2) + \alpha x(2-t)$
$= \alpha y(t) \Rightarrow$ {homogeneous}

$\Rightarrow$ linear.

4. $x(t) < \alpha$
$y(t) = x(t-2) + x(2-x) \leq 2\alpha$; Hence system is stable

5. $x(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$
$= y(t-t_0)$

Hence it is invariant

(b) $y(t) = \begin{cases} 0 & , t \leq 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases}$

1. $y(2) = x(2) + x(0) \Rightarrow$ Has memory
2. $\Rightarrow$ ~~no~~ causal
3. $y(t_1) + y(t_2) = x(t_1) + x(t_1-2) + x(t_2) + x(t_2-2)$
$= y(t_1) + y(t_2) \Rightarrow$ {superposition}

Linear system:

4) $x(t) < \alpha$

$y(t) \leq 2$

stable

5) $y(t-t_0) = \cancel{x(t)} + x(t-t_0) + x(t-t_0 \mp 2)$

$y(t) = x(t-2)$

c) $y[n] = x[2-n] - 2x[n-8]$

$y(2) =$ memory

$\Rightarrow$ non causal.

$\Rightarrow$ Linear

$\Rightarrow$ stable

$\Rightarrow$

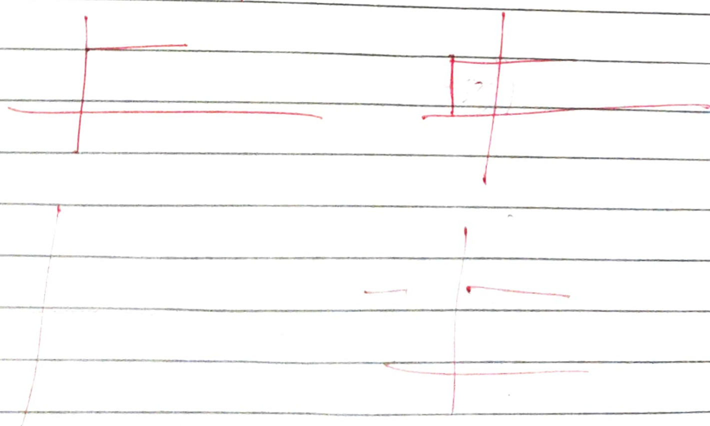
$y[n-n_0] = x[2-n+n_0] - 2x[n-n_0-8]$

time varying.

d) $\dfrac{x[n-1] + x[1-n]}{2}$ $\Rightarrow$ has memory

$\Rightarrow$ ~~has~~ non causal

$\Rightarrow$



$x_1(t) \to x_1(t+2) \to x_1(-t+2)$

d) $\frac{x[n-1]+x[1-n]}{2}$: $\frac{x[n-n_0-1]+x[1-n+n_0]}{2}$

$x[n-n_0]$

$y_1(t) = \cancel{x[n-1]}\ \frac{x[n-n_0-1]+x[1-n+n_0]}{2}$

$x[n] \to x[n-n_0-1]$

$= x[n-n_0-1]$

$x[n] \to x[n+1] \to x[1-n]$

$x[n-n_0] \to x[n-n_0+1] \to x[-n-n_0+1]$

$\frac{x[n-n_0-1]+x[-n-n_0+1]}{2}$ ⇒ time variant

e) $y[n] = \begin{cases} x[n] & , x \geq 1 \\ 0 & , n = 0 \\ x[n+1] & , n \leq -1 \end{cases}$

⇒ has no memory

⇒ non causal

⇒ ~~l~~ linear

$y[n-n_0] = \begin{cases} xt \\ \\ \end{cases}$

$x[n-n_0]$

$$\Rightarrow y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$$

1. memory less
2. ~~ca~~ non causal
3. linear system
4. stable

5. $Re\left(e^{j\pi n/4}\, x[n]\right) = y[n]$

$\Rightarrow$ memory less.

$\rightarrow$ ~~non~~ causal

$\Rightarrow$ ~~$Re(\cos\theta + j\,s$~~ $Re\left(\left(\cos\left(\frac{\pi n}{4}\right) + j\sin\left(\frac{n\pi}{4}\right)\right)\left(x[n] + jx_I[n]\right)\right)$

$\Rightarrow Re\left[x_R[n]\cos\frac{\pi n}{4} - x_I[n]\sin\frac{n\pi}{4}\right]$

$y_1(t) = x_R^1[n]\cos\frac{\pi n}{4} - x_I^1[n]\sin\frac{\pi n}{4}$

$y_2(t) = x_R^2[n]\cos\frac{\pi n}{4} - x_I^2[n]\sin\frac{\pi n}{4}$

$y_1[n] + y_2[n] \rightarrow$ superimposed

α can be complex as well.

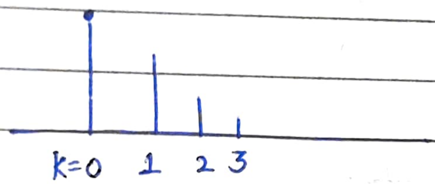
$\delta[n] \longrightarrow h[n]$

$x[n] \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \longrightarrow$ convolution sum

$x[n] * h[n]$
↓
convolution operator

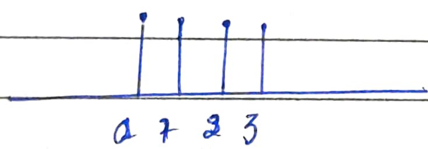
$x[n] = \alpha^{n} u[n]$

$h[n] = u[n]$
↓
Impulse response.

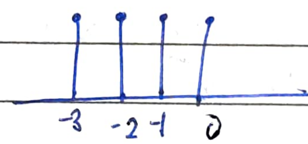


$\underline{\alpha < 1}$

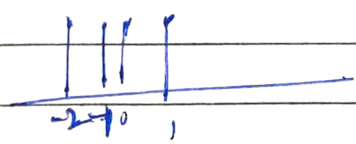
$h[k] =$



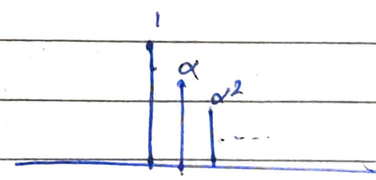
$\longrightarrow h[-k]$



$h[1-k] =$



multiply →

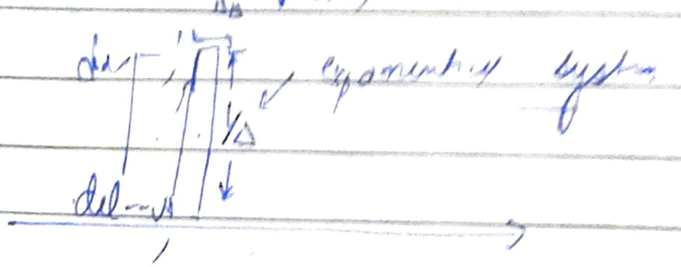


$y[n] = \sum_{k=-\infty}^{\infty} \alpha^{k} u[k]\, u[n-k]$
↓
1 for only positive

$= 1 + 7/ = 0$

The final outcome = $\frac{1-\alpha^{n+1}}{1-\alpha}$

$$y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u[n]$$



$$x(t) = \sum_{n=0}^{\infty} \delta(t-2\beta)$$

Q5 $\cancel{x[0] = \ldots}$ $\cancel{x[0] = \ldots}$

$x[0] =$

lim $[t]$ > 8

$$\delta(t) \xrightarrow{S} h[t]$$

$$x(t) \longrightarrow y(t)$$

~~Linear time signal~~

Linear time — depending

$\cancel{\Delta t}$ $\Delta = 0$

$$y(t) = \int x(\tau)$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau} \longrightarrow \text{system}$$

$x(t) * h(t-\tau)$

Varying

we can write any integral into other form

Note: $h(t) \rightarrow$ impulse response
Taking unit step

$u(\,) \xrightarrow{h} g(\,)$

Q. $x(t)\, e^{-at}$ find $y(t)$

$f(t) = u(t)$

$u(\tau) = \begin{cases} 1, & \tau \geq 0 \\ 0, & \tau < 0 \end{cases}$

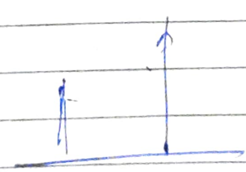
$$\int_0^t e^{-a\tau}\, d\tau = \frac{e^{-a\tau}}{-a}$$

$- 2a e^{-?}$

$\Rightarrow \; 2e^{-2a\tau}$

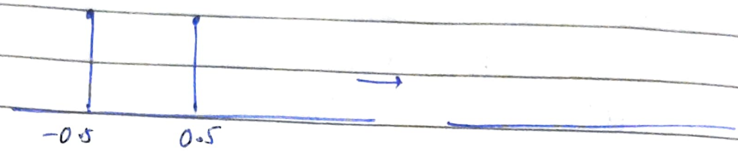
$$y(t) = \left(\frac{1 - e^{-at}}{a}\right) = \frac{1-e^{-at}}{a}$$

Q. $x(t)$:

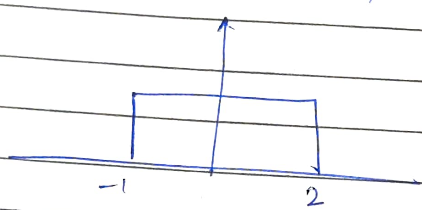


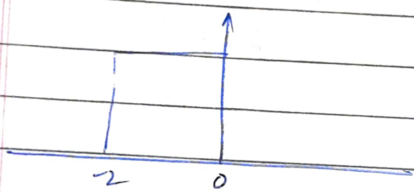
-2

Convulution breaks an input into impulses and they add their outputs.



$x(t) =$ rectangular ~~pulse~~ pulse.





- Discrete time Convolution:-

$$x[n] \xrightarrow{h[n]} y[n] + \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

Continuous time convolution:-

$$x(t) = y(t) + \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

Properties

1) $$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)$$

$$t - \tau = p$$

$$y(t) = \int_{-\infty}^{\infty} x(t-p)\, h(p)\, dp$$

$$= \int_{-\infty}^{\infty} h(p)\, x(t-p)\, dp = \cancel{x(t)}\ h(t) * x(t)$$

$$h(t) \rightarrow \boxed{} \rightarrow y(t)$$ { Commutative property }

Associative :→ $x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t)$

$$= x(t) * h_1(t) * h_2(t)$$

{Linear Convulation}

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

$$\downarrow$$

$$y(t) = (x(t) * h_1(t)) * h_2(t)$$

For LTI systems.

$$x_1(t) \longrightarrow \boxed{h_2(t)} \longrightarrow \boxed{h_1(t)} \longrightarrow y(t)$$

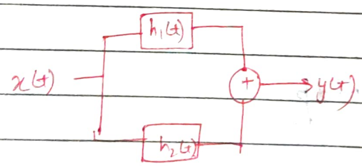
$$\downarrow$$

$$y(t) = (x_1(t) * h_2(t)) * h_1(t)$$

$$x_1(t) \longrightarrow \boxed{h_2(t) * h_1(t)} \longrightarrow y(t)$$

Distributive property :→

$$x(t) * (h_1(t) + h_2(t)) = x_1(t) * h_1(t) + x_2(t) * h_2(t)$$

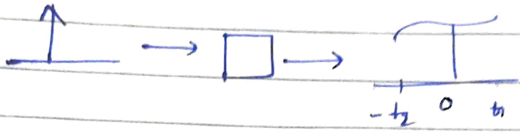


$$\downarrow$$

$$x(t) \longrightarrow \boxed{h_1(t) + h_2(t)} \longrightarrow y(t)$$

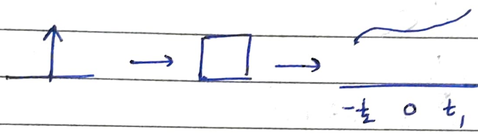
2) Memory Less :→

$h(t) = K\delta(t)$  {memory less}

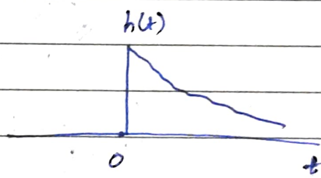


) Causality :→

for the following response; system is causal :→

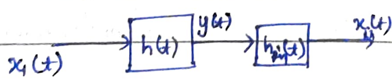


$h(t) = 0 \{t < 0\}$



causal → shouldn't depend on future.

) Invertibility :→



$y(t) = x(t) * h(t)$
$y_{in}(t) = x(t) * h(t) * h_{in}(t)$

$x(t) = x(t) * (h(t) * h_{in}(t))$

$\boxed{x(t) = x(t) * \delta(t)}$

$$x(t) * \delta(t) = x(t) * (h(t) * h_{in}(t))$$

$$\boxed{\delta(t) = h(t) * h_{in}(t)} \longrightarrow$$ If this is true system is invertible.

Stability :-

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n] y[n-k]$$

$$|x[n]| < C$$

$$y[n] = \left| \sum_{k=-\infty}^{\infty} x[k] h[n-k] \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |x[k]| \, |h[n-k]| \leq C \left| \sum_{k=-\infty}^{\infty} |h[n-k]| \right| < \infty$$

Hence $\left| \sum_{k=-\infty}^{\infty} |h[n-k]| \right| < \infty$

$$\Rightarrow \sum_{k=-\infty}^{\infty} |h[n]| < \infty \quad \{\text{for discrete}\}$$

$$\Rightarrow \int_{-\infty}^{\infty} |h(t)| \, dt < \infty \quad \{\text{for continuous}\}$$

Q $h(t) = e^{-t} \cos 2t \; u(t)$ stable or not

$\Rightarrow \int_0^\infty e^{-t} |\cos 2t| \, dt$

$\Rightarrow \cancel{e^{-t}} \; \underline{|\cos 2t| \, e^{-t}}_{\,0} - \int e^{-t} \frac{|\sin 2t|}{2} \, dt = I$

→ stable

Q $h(t) = r^n \cos\left(\frac{\pi}{4} n\right) u(t)$

Q $e^{st} = \quad s = \cancel{\alpha} + \beta j$

$\Rightarrow e^{\sigma t} e^{\beta j t}$

$x(t) = e^{st} \longrightarrow \boxed{} \longrightarrow y(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \, h(t-\tau) \, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{s\tau} \, h(t-\tau) \, d\tau$$

$$= \int_{-\infty}^{\infty} e^{s\tau - st + st} \, h(t-\tau) \, d\tau = \int_{-\infty}^{\infty} e^{st} e^{-st} e^{s\tau} \, h(t-\tau) \, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} e^{-s(t-\tau)} \, h(t-\tau) \, d\tau$$

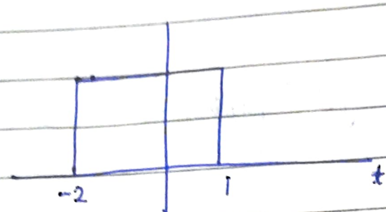
$$= e^{st} \int_{-\infty}^{\infty} e^{-sp} \, h(p) \, dp = H(s) \, e^{st}$$

{Laplace}

$e^{st} \longrightarrow$ Eigen function.

$H(s) \longrightarrow$

Q



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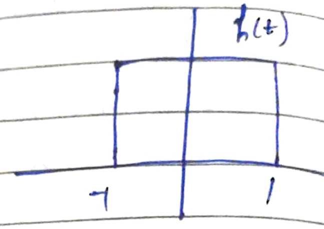
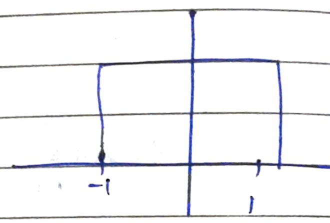
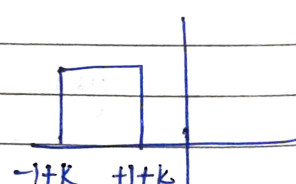


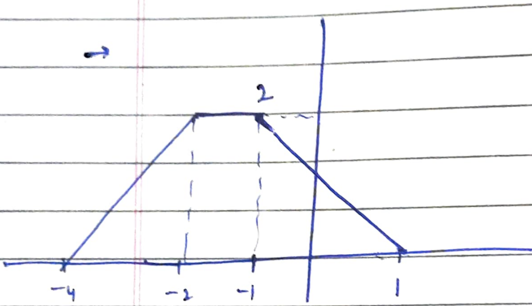
$\downarrow h(-t)$



$\downarrow h(k-t)$

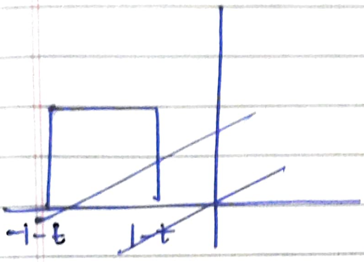


$\rightarrow$

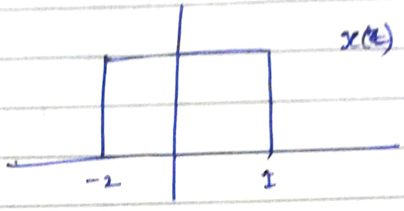


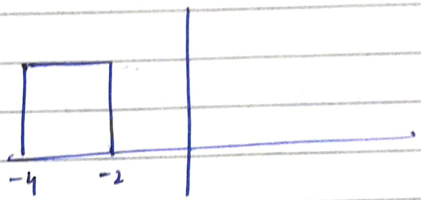
$$y(t) = \int_{-\infty}^{\infty} x(t)\, h(t-\tau)\, d\tau$$

$$h(\tau) \xrightarrow[\text{right by } t]{\text{shift}} h(\tau+t) \xrightarrow{\text{Invert}} h(-\tau+t)$$

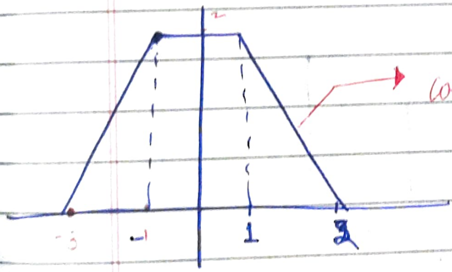


x(t)





y(t)



convulution

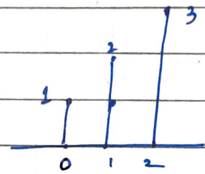
removing → dessimation {reduce signal}

adding → interapolation {increase signal}

(2) b) $y[n] = n\,x[n]$, for $n=0$, the value is always 0, so we can't get $x[n]$ back

c) $y[n] = x[n]\,x[n-1]$, If $x[n-1], x[n+1] = 0$ then all values are equal then we loose $x[n]$

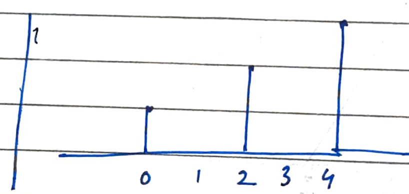
d) $y[n] = x(2n)$



dessimation

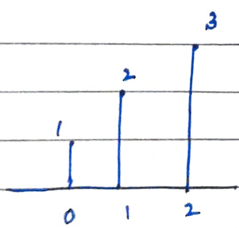
for odd values of n, $x[n]$ is lost.

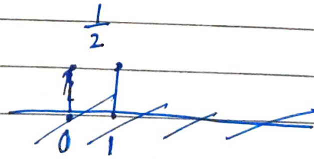
e) $y[n] = \begin{cases} x[n/2], & n = \text{even} \\ 0, & n = \text{odd} \end{cases}$



$z[n] = y(2n]$, hence it is invertible.

f) $y[n] = \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^{n-k} x[k]$





1 $\quad \frac{1}{2} \times 1$

Special accumulator.

$$y[n-1] = \sum_{k=-\infty}^{n-1} \left(\frac{1}{2}\right)^{n-k-1} x[k-1]$$

$$y[n] - y[n-1] = \left(\frac{1}{2}\right)^{n-n} x[n]$$

$$y[n] - y[n-1] = x[n] \Rightarrow \boxed{y[n] = x[n] - x[n-1]}$$

Inverse.

discounted

└• past should be weighted.

3

$$x(t) = e^{j2t} \longrightarrow y(t) = e^{j3t}$$

$$x(t) = e^{-j2t} \longrightarrow y(t) = e^{-3jt}$$

a) $x_1(t) = \cos 2t$ { linear time system }

$$x(t) = \cos 2t + (\sin 2t)j \longrightarrow \cos 3t + (\sin 3t)j$$

$$\cos 2t - \sin 2t j \longrightarrow \cos 3t - \sin 3t j$$

So

$$\cos 2t \longrightarrow \cos 3t$$

Hence

$$x(t) = \cos 2t \xrightarrow{S} \cancel{\sin 3t} \cos 3t$$

b) $$\cos\left(2\left(t-\frac{1}{2}\right)\right) \xrightarrow{S} \cos\left(3\left(t-\frac{1}{2}\right)\right)$$

$$\longrightarrow \cos\left(3t - \frac{3}{2}\right)$$

(4) $x[n] \xrightarrow{S} y[n]$

~~x[a~~ $y[n] = x[n\alpha]$

$= \alpha x[n]$

$y[n\alpha] = \alpha y[n]$

let $n = 0$

$y[0] = \alpha y[0]$

This property of homogeniety is true for all values of α.

Hence $y[0] \equiv 0$

$y[n] = S(x[n])$

$y[\alpha n] = \alpha S(x[n])$

$\neq$ If

$\cancel{y_1[n_1] + y_2[n_2] = S(x_1[n_1] + x_2[n_2])}$

$y_1[n] = S(x_1[n])$

$$y_1[n] + y[n_2] = x[n_1] + x[n_2]$$

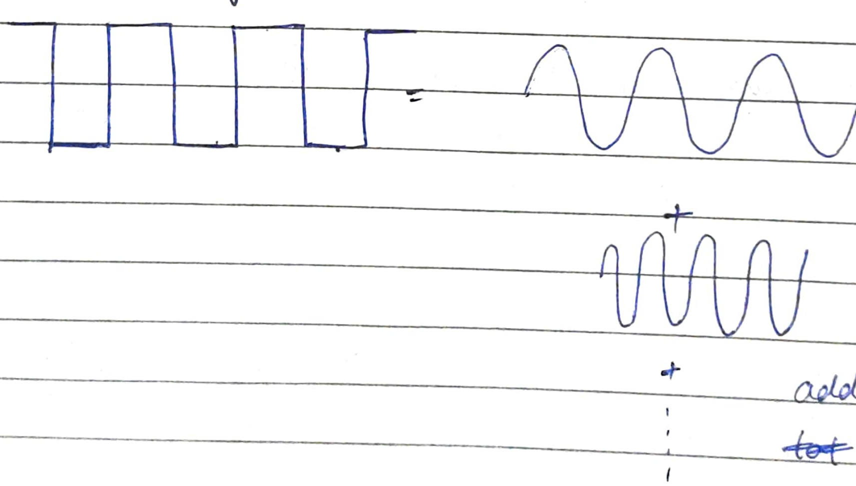
$$x[n] \longrightarrow y[n]$$
$$x[n] \longrightarrow y[n]$$
$$2x[n] \longrightarrow 2y[n]$$

$$2y[n] = y[n]$$
$$\boxed{y[n] = 0}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$ → complex functiont is eigen f^n of LTI system

$$e^{jk\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow e^{j\omega t} \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

Consider a periodic signal with period P.



add these signals ~~tot~~ together to make the signal

x(t) is periodic with period T. Frequency of signal → $f = \frac{1}{T}$.

frequency = $f, 2f, \ldots$

any signal with frequency f can be written as sum of signals with frequency $f, 2f, 3f, \dots$

Frequency components are orthogonal

$x(t)\ y(t)$ are orthogonal $\Rightarrow \int_{-\infty}^{\infty} x(t)\, y(t)\, dt = 0$

$e^{jk_1\omega_0 t} \quad e^{-jk_2\omega_0 t}$

$$\Rightarrow \int_0^T e^{j(k_1-k_2)\omega_0 t}\, dt \quad \{\text{time period } T\}$$

$$\Rightarrow \frac{e^{j(k_1-k_2)\omega_0 T} - 1}{(k_1-k_2)\omega_0 T} = 0$$

$$\Rightarrow \frac{e^{j(k_1-k_2)\cancel{\omega_0} 2\pi} - 1}{(k_1-k_2)\frac{2\pi}{T}} = 0$$

Over a time period T, signals are complex.

$$x(t) = \sum_{t=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \quad \omega_0 = f, 2f, 3f, \dots \longrightarrow \text{synthesis equation.}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt \longrightarrow \text{Analysis Equation}$$

$$a_0 = \frac{1}{T}\int_T x(t)\, dt.$$

Fourier Transform

$x(t) = \cos\omega_0 t \Rightarrow \boxed{T_0 = \frac{2\pi}{\omega_0}}$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$; $a_k \longrightarrow$ complex.
$\omega_0 \longrightarrow$ from $-\infty$ to ∞.

$$\cos\omega_0 t = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0}$$

$$x(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

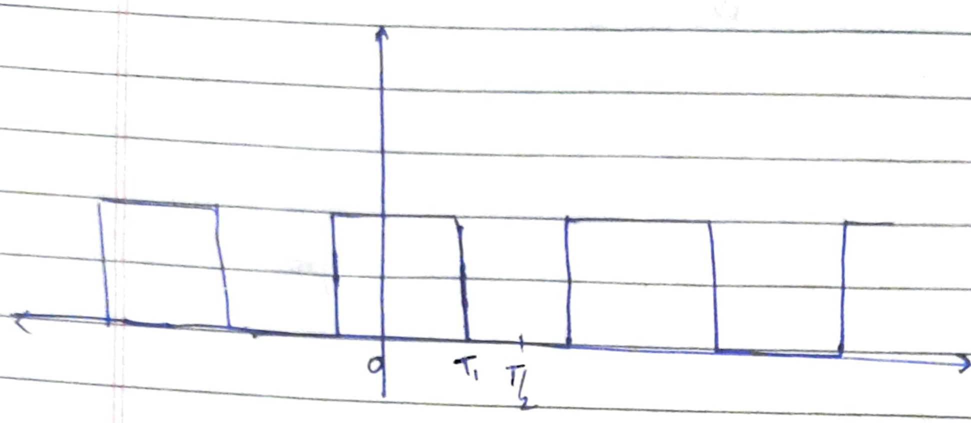
$a_1 = \frac{1}{2}$, $a_{-1} = \frac{1}{2}$; $a_k = 0$, $k \neq -1, 1$

$$\text{period} = \frac{2\pi}{\omega_0}$$

$$x(t) = \sin(\omega_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$x(t) = \cos\left(\omega_0 t + \frac{\pi}{4}\right) \Rightarrow \frac{\cancel{1}}{\cancel{\sqrt{2}}} \quad \frac{\cos\omega_0 t}{\sqrt{2}} - \frac{\sin\omega_0 t}{\sqrt{2}}$$

$$\Rightarrow \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2\sqrt{2}}\right) - \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2\sqrt{2}j}\right)$$



$$x(t) = \sum a_k e^{kj\omega_0 t}$$

$$a_k = \frac{1}{T} \int_{T} x(t)\, e^{-jk\omega_0 t}\, dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} 1\, e^{-jk\omega_0 t}\, dt$$

$$\Rightarrow \frac{1}{T} \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_{-T/2}^{T/2} \qquad \cos\theta + i\sin\theta$$

$$\Rightarrow \frac{1}{\cancel{T}} \left[\frac{e^{-jk\frac{2\pi}{T} T/2} - e^{jk2\pi/2}}{-jk\frac{2\pi}{\cancel{T}}} \right]$$

$$\Rightarrow \frac{e^{-j\omega_0 2\pi} - e^{+j\omega_0 2\pi}}{-2\pi kj}$$

$$= \frac{\sin(\omega_0 kT)}{\pi k}.$$

Suppose $x(t)$ is real $\Rightarrow$ $x(t) = x^*(t)$

$$\boxed{a_k = a^*_{-k}}$$

$$\sum a_k e^{jk\omega_0 t} = \sum a^*_k e^{-jk\omega_0 t}$$

$$= \sum a^*_{-k} e^{jk\omega_0 t}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_k \left(\cos(k\omega_0 t) + j \sin(\omega_0 t)\right)$$

$\cancel{a_k = A e^{j}}$ $\qquad a_k = A_k e^{j\phi_k}$,

$$\cancel{x(t) = \sum_{k=-\infty}^{\infty} A_k e^{j}(\omega_0 t + \phi_k) + j\sin(k\omega_0}$$

$$x(t) = \sum_{k=-\infty}^{\infty} A_k e^{j(k\omega_0 t + \phi_k)}$$

$$\cancel{x(t) = \sum_{k=-\infty}^{\infty} A_k (\cos(j}$$

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \left(\cos(k\omega_0 t + \phi_k) + j\sin(k\omega_0 t + \phi_k)\right)$$

$$\boxed{A_k e^{j\phi_k} = A_{-k} e^{-j\phi_k}}$$

$$\cancel{x(t) = A_0 + \sum_{k=1}^{\infty} (A_k + A_{-k}) \cos(k\omega_0 t + \phi_k) + j}$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k e^{j\phi_k} e^{jk\omega_0 t} + \cancel{\sum} A_{-k} e^{j\phi_{-k}} e^{-jk\omega_0 t} \right)$$

$$= A_0 + \sum_{k=1}^{\infty} b_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} c_k \sin(k\omega_0 t)$$

$$\Rightarrow \quad b_k = a_k + a_{-k} \qquad c_k = j(a_k - a_{-k})$$

$$x(t) = a_0 + \sum_{n=-\infty}^{\infty} a_k \cos k\omega_0 t + \overset{a_k}{j} \sin(\omega_0 t k) + a_{-k} \cos k\omega_0 t - a_{-k} \overset{k}{\sin \omega_0 t}$$

$$= a_0 + \sum_{n=-\infty}^{\infty} (a_k + a_{-k}) \cos k\omega_0 t + (a_k - a_{-k}) j \sin \omega_0 t k$$

$$\Rightarrow a_0 + \sum_{n=-\infty}^{\infty} b_k \cos k\omega_0 t + c_k \sin \omega_0 t$$

Synthesis Equation →

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

for $x(t) \in \mathbb{R}$

$$a_k^* = a_{-k}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(a_k e^{jk\omega_0 t} + a_{-k}^* e^{-jk\omega_0 t} \right)$$

$$p + p^* = 2Re(p)$$

$$= a_0 + 2\sum_{k=1}^{\infty} Re\{a_k e^{jk\omega_0 t}\} \qquad a_k = A_k e^{j\theta_k}$$

$$= a_0 + 2\sum_{k=1}^{\infty} Re\{A_k e^{j\theta_k} e^{jk\omega_0 t}\}$$

$$= a_0 + 2\sum_{k=1}^{\infty} \underbrace{A_k}_{\text{diff. amp}} \cos(\overbrace{k\omega_0 t}^{\text{diff. freq.}} + \underbrace{\theta_k}_{\text{diff. Amp}})$$

$$a_k = p_k + j q_k$$

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} Re\{(p_k + j q_k)(\cos(k\omega_0 t) + j\sin(k\omega_0 t))\}$$

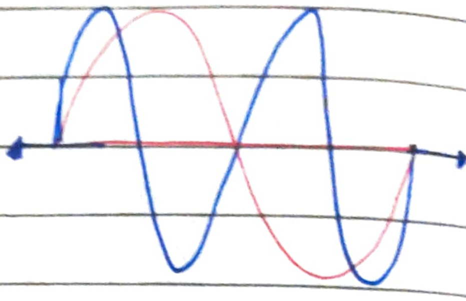
$$= a_0 + 2\sum_{k=1}^{\infty} p_k \cos(k\omega_0 t) \mp q_k \sin(k\omega_0 t)$$

$$= a_0 + 2\sum_{k=1}^{\infty} (p_k \cos(k\omega_0 t) - q_k \sin(k\omega_0 t))$$

$x(t) \in \mathbb{R}$

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left(b_k \cos(k\omega_0 t) + c_k \sin(k\omega_0 t) \right)$$

$$\frac{1}{T} \int x(t) \cos(k\omega_0 t)\, dT$$



$$\left\{ \frac{1}{T} \int_T x(t) \cos(k\omega_0 t)\, dt = \frac{b_k}{2} \right\}$$

$$\left\{ \frac{1}{T} \int_T x(t) \sin(k\omega_0 t)\, dt = \frac{c_k}{2} \right\}$$

- The k^{th} component Harmonic
  for $k=0$, DC component
  for $k=1$, fundamental Harmonic

CONVERGENCE

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$$

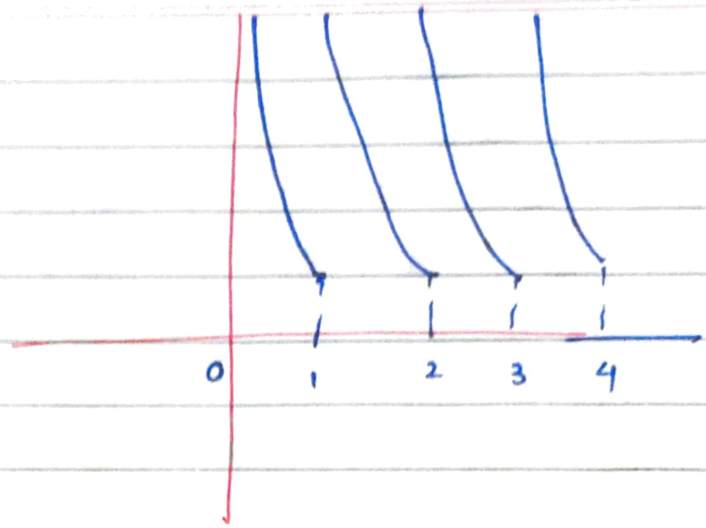
$$e_N(t) = x(t) - x_N(t)$$

E_N : Energy in error $\Rightarrow \int_T |e_N(t)|^2 dt \longrightarrow 0$.

As $N \to \infty$, $E_N \to 0$ {for convergence}

Dirichlet's Condition :-

(I) $\int_T |x(t)|\, dt < \infty$



$x(t) = \frac{1}{t}, \quad 0 \le t \le 1$

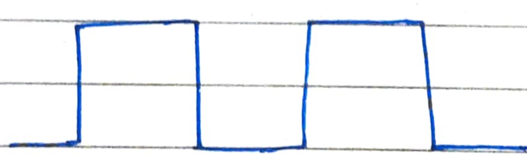
It is not absolutely Integrable.

(II) x(t) must have finite no of maxima & minima.

$x(t) = \sin\left(\frac{2\pi}{t}\right), \quad 0 < t \le 1.$

{near to $t \to 0$, the pulse oscillates}

(III) x(t) should have finite discontinuity. {over the period}



Ripple formed at ~~disc~~ discont is calle ~~Gibbs phenomenon~~
Gibbs phen

(1) Linearity

$x(t) \xrightarrow{F} a_k$

$y(t) \xrightarrow{F} b_k$

$y(t) + x(t) \xrightarrow{F} a_k$

(II) Time shifting:

$$x(t) \xrightarrow{F.S.} a_k$$
$$x(t-t_0) \xrightarrow{F.S.} a_k e^{-jk\omega_0 t}$$

(III) Time reversal:

$$\cancel{x(t)}\; x(-t) \xrightarrow{F.S.} a_{-k}.$$

(IV) Multiply

$$x(t) = a_k$$
$$y(t) = b_k.$$

$$z(t) = x(t)\,y(t) \qquad \{\text{same period } T\}$$

$$x(t) = \sum_{t=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$y(t) = \sum_{t=-\infty}^{\infty} b_k e^{jk\omega_0 t}$$

$$\cancel{h(t)}\; h_k = \frac{1}{T}\int_T z(t)\, e^{-jk\omega_0 t}\, dT.$$

$$= \frac{1}{T}\int_T x(t)\,y(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_T \left(\sum_{t=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right)\left(\sum_{t=-\infty}^{\infty} b_k e^{+jk\omega_0 t}\right) e^{-jk\omega_0 t}\, dt$$

$$\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\frac{1}{T}\int_T a_l b_{\cancel{n}m}\, e^{(l+m)j\omega_0 t}\, e^{-jk\omega_0 t}$$

$$\Rightarrow \sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\frac{a_l \cancel{b_m}}{2}\underbrace{\int_T e^{j\omega_0 t(l+m)^{-k}}\,dt}$$

$\longrightarrow = 0$ for $l+m-k \neq 0$

$\longrightarrow = T$ for $\underline{\underline{l+m=k}}$.

$$\Rightarrow \sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\frac{a_l b_m T \cancel{dt}}{2}$$

$$h_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l} \rightarrow \text{convolution.}$$

$$\int_{-\infty}^{\infty} x(\tau)y(t-\tau)\,d\tau \xrightarrow{F.S.} a_k b_k$$

Complex conjugate property:→

$$x(t) \longrightarrow a_k$$

$$x^*(t) \longrightarrow a_{-k}^*$$

$x(t)$ is even.

$x(t) = x(-t)$ $\qquad \Rightarrow x(-t) = \sum a_k e^{-jk\omega_0 t}$

$$\boxed{a_k = a_{-k}}$$

$$\int_0^T |x(t)|^2 dT = \int_0^T \left|\sum a_k e^{jk\omega_0 t}\right|^2 dt$$

$$= \int_0^T \sum_k |a_k|^2 |e^{jk\omega_0 t}|^2 \cancel{+ 2\int_0^T \int_0^T \sum |a_k||e^{j(k+l)\omega_0 t}|}$$

$$\Rightarrow \sum |a_k|^2 = \underbrace{\frac{1}{T}\int_0^T |x(t)|^2 dt}\ \cancel{+2}$$

$|a_k|^2 \rightarrow$ average power in ~~#~~ k^{th} harmonic

- $x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$e^{jk\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} h(\tau) e^{jk\omega_0 (t-\tau)} d\tau$$

$$= e^{jk\omega_0 t} \int_{-\infty}^{\infty} h(\tau) e^{-jk\omega_0 \tau} d\tau$$

$$= e^{jk\omega_0 t} H(k\omega_0)$$

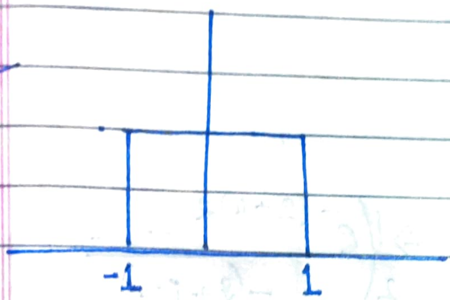
$x(t) \rightarrow \boxed{\text{LTI}} \rightarrow y(t) = \sum_{k=-\infty}^{\infty} a_k H(k\omega_0)\, e^{jk\omega_0}$

$x(t)$ → periodic.

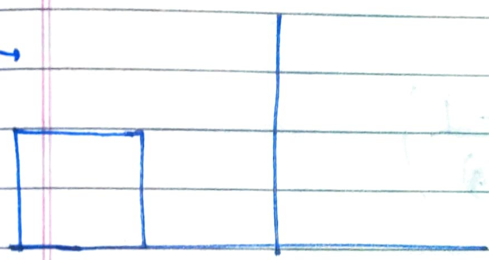
$y(t)$ → periodic with period T.

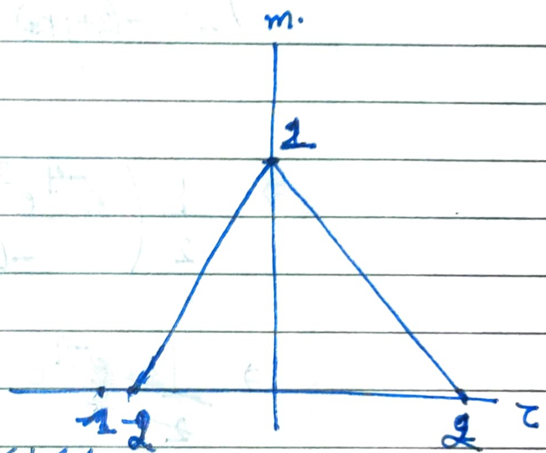
Fourier coefficients:-
$a_k H(k\omega_0)$

Q



$x(t) * x(t)$





$f(t) = 1, \; -1 < t < 1$

$f(t-\tau) = 1, \; -1 < t-\tau < 1$

τ $\quad x(\tau)$ $\quad x(t) = u(t+1) - u(t-1)$

$x(t-\tau) = u(t+1-\tau) - u(t-1-\tau)$

Ques. $x(t) = e^{-2t} \quad 0 \le t < 2$

periodic

$$a_k = \frac{1}{2}\int_{-\infty}^{\infty} e^{-2t} e^{-jkt\omega_0} dt$$

$$= \frac{1}{2}\int_{0}^{2} e^{-(2+jk)t} dt$$

$$= \int \frac{1}{2}\left(\frac{e^{-(2+jk\omega_0)t}}{-(2+jk\omega_0)}\right) \Rightarrow \frac{1}{2}\left(\frac{e^{-(2+jk)2} - 1}{-(2+jk)}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{e^{-4} e^{2jk\omega_0} - 1}{-(2+jk\omega_0)}\right)$$

$$\Rightarrow \cancel{\frac{1}{2} e^{-4} e}$$

$$= \frac{1}{2}\left(\frac{1 - e^{-4} e^{2\pi jk}}{2+jk}\right)$$

$$\Rightarrow \cancel{\frac{1}{2}\left(1 - e^{-4}(\cos \pi k + j\right.}$$

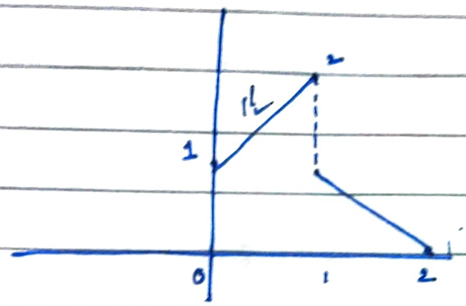
$$= \frac{1}{2}\left(\frac{1 - e^{-4}(\cos \pi k + j\sin \pi k)}{2+jk}\right)$$

$$\Rightarrow \frac{e^{-4}-1}{-(4+2jk)} = \frac{1-e^{-4}}{2jk+4}$$

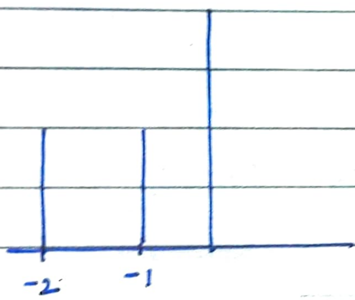
Tutorial-3

1

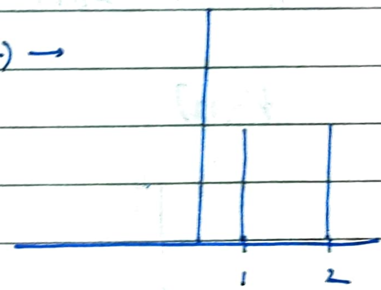
$x(t)$



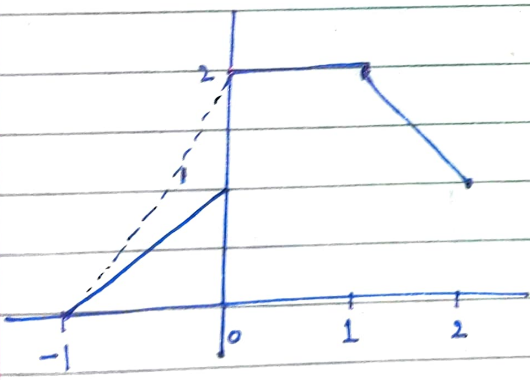
$h(t) = \delta(t+2) + \delta(t+1)$

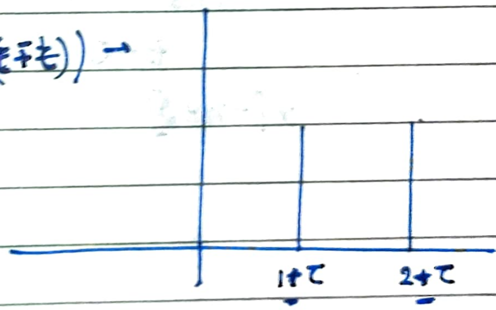


$h(-t) \rightarrow$



$h(-(\tau - t)) \rightarrow$





$\tau \rightarrow 1$

$\tau = -1$

$$y = \left(\frac{2-1}{1-0}\right)x + 1$$

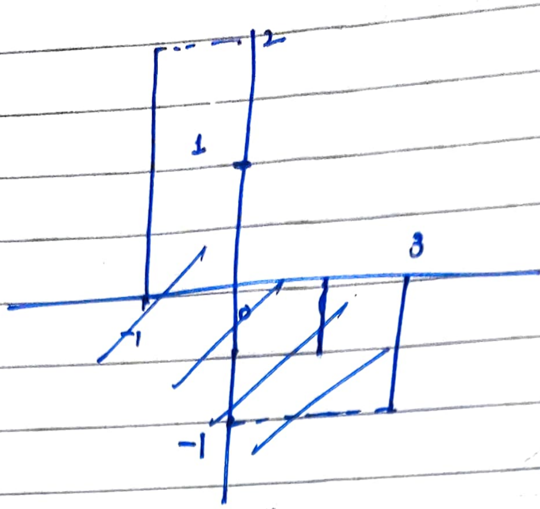
$\frac{1}{2} =$

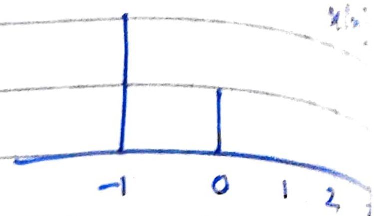
$$y = x+1$$

$$y = \left(\frac{1-2}{2-1}\right)x + 1$$

$$y = -x+1$$

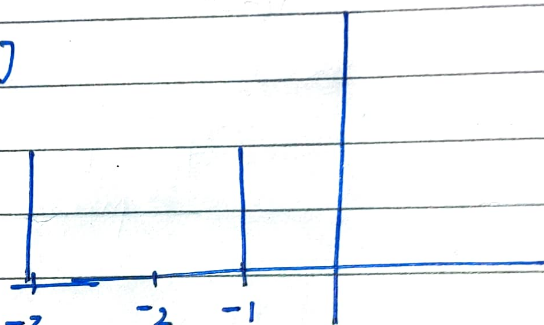
② $x[n] = \delta[n] + 2\delta[n+1] - \delta[n-3]$





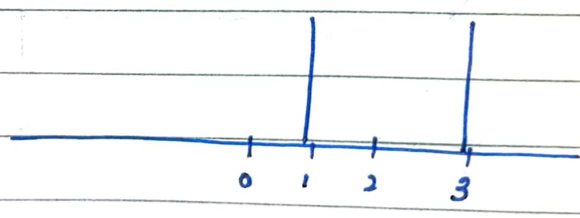
$h[n+2] = 2\delta[n+3] + 2\delta[n+1]$

$h[n+2]$



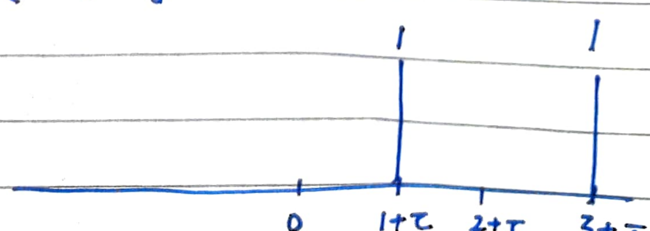
$t - \tau$

$x[-n+2]$



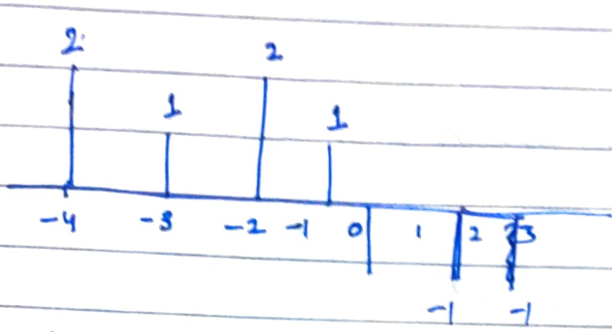
$x[-(n-t)+2]$

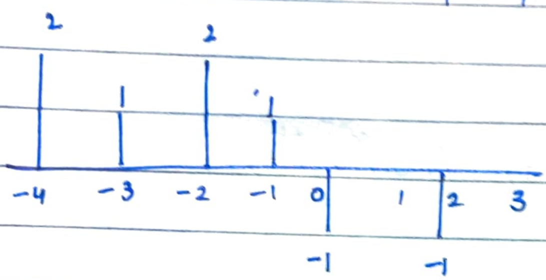
$h[t-n+2]$



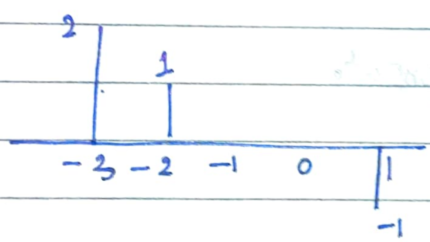
$3+\tau = -1$

$\tau = -4$





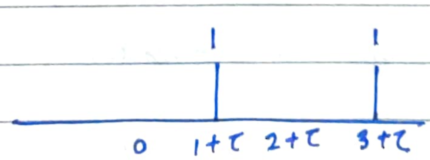
(b) $x[n+2] = \delta[n+2] + 2\delta[n+3] - \delta[n-1]$



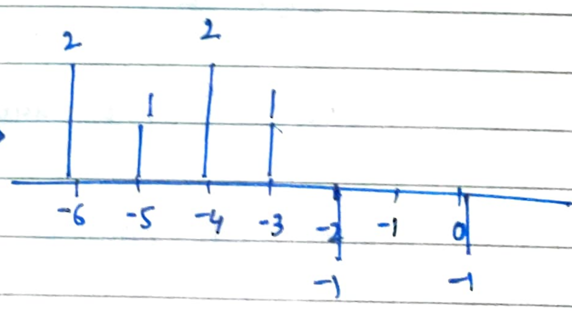
$h[n+2] = 2\delta[n+3] + 2\delta[n+1]$

$x[n+2] * h[n+2]$

$h[\tau - n + 2]$



$\Rightarrow$



$3 + \tau = -3$ $\quad$ $1 + \tau = 1$

$\boxed{\tau = -6}$ $\quad$ $\tau = 0$

③ a) $h(n) = \left(\frac{-1}{2}\right)^n u[n] + (1.01)^n u(n-1)$

→ ~~If~~ since for the system is time invariant the following is true.

$$x(n) \longrightarrow h(n)$$
$$h(n+t) \longrightarrow h(n+t)$$

$$h(n+t) = \left(\frac{-1}{2}\right)^{n+t} u(n+t) + (1.01)^{n+t} u(n-1+t)$$

Hence for all values of t (n), the response of system depends on $n+t$ & $n+t-1$ only, hence system is causal.

Assume a bounded input.

$$x[n] \leq \alpha$$

$$h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u(n-1)$$

~~for~~

~~since $a^x u$~~

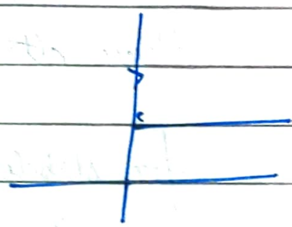
since a^x is an always increasing function for $a>1$

Hence the response $h[n]$ is not bounded.

Hence it is not stable.

b) $h(t) = e^{-6t}\, u(3-t)$

$u(t) =$



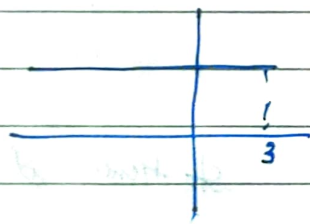
as we know a linear time invariant system is causal if for all values of $t<0$, the response of the system is 0.

$u(-t)$

~~for~~

Hence the system is non-causal.

$u(-(t-3))$



$$\int_{-\infty}^{\infty} e^{-6t} u(3-t)\, dt$$

$$\int_{-\infty}^{3} e^{-6t}\, dt \Rightarrow \frac{e^{-6t}}{-6} \Rightarrow \frac{e^{-18t}}{-6} - 0$$

$$= \frac{e^{-18t}}{+6} < \infty$$

Hence the system is stable.

(c) $h(t) = te^{-t}u(t)$

Since the $u(t)$ function is 0 for $t \leq 0$.

Hence $h(t)$ is a causal function.

for stability

$$\int_{-\infty}^{\infty} te^{-t}dt = \int_{0}^{\infty} te^{-t}dt \Rightarrow -te^{-t} + \int e^{-t}dt$$

$$\Rightarrow -te^{-t} - e^{-t}\Big)_0^{\infty}$$

$$\Rightarrow (0-0) - (-1)$$

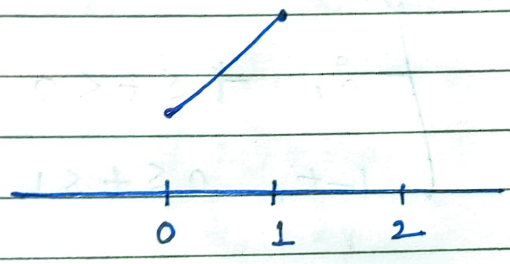
$$= 1$$

~~So~~ Hence it is stable.

① $x(t) * \overset{\delta(t+\tau)}{\cancel{h(t)}} = \cancel{\int} \; x(t+\tau)$

$$\boxed{x(t) * \delta(t+\tau) = x(t+\tau)}$$

$$x(t) = \begin{cases} t+1, & 0 \leq t \leq 1 \\ 2-t, & 1 < t \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$



$$h(t) = \delta(t+2) + \delta(t+1)$$

$$x(t) * h(t) = x(t) * \left(\delta(t+2) + \delta(t+1)\right)$$

$$= x(t+2) + x(t+1)$$

$0 \leq t+2 \leq 1$

$-2 \leq t \leq -1$

$$x(t+2) = \begin{cases} \overset{+2}{t+1}, & -2 \leq t \leq -1 \\ 2-(t+2), & -1 \leq t \leq 0 \\ 0 & \text{else where} \end{cases}$$

$1 < t+2 \leq 2$

$-1 < t \leq 0$

$$x(t+2) = \begin{cases} t+3, & \cancel{1}\ -2 \leq t \leq -1 \\ -t, & -1 \leq t \leq 0 \\ 0 & \text{else where} \end{cases}$$

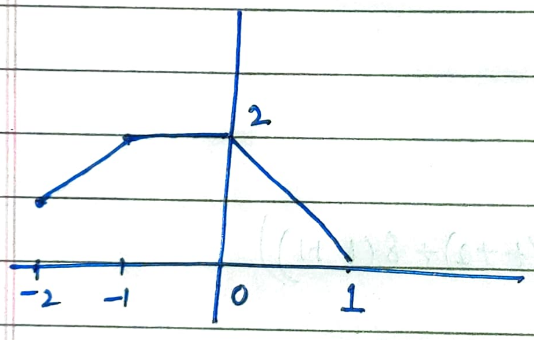
$0 < t+1 \le 1$ $2-t-1$

$1 < t+1 < 2$

$$x(t+1) = \begin{cases} t+2, & -1 < t \le 0 \\ 1-t, & 0 < t \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

$t+2+t+3$

$$x(t+2) + x(t+1) = \begin{cases} t+3, & -2 < t < -1 \\ 2, & -1 \le t < 0 \\ 1-t, & 0 \le t \le 1 \end{cases}$$



4

2 a) $x(n) = \delta[n] + 2\delta[n+1] - \delta[n-3]$

$h(n] = 2\delta[n+1] + 2\delta[n-1]$

$x[n] * h[n] = 2\,x[n+1] + 2\,x[n-1]$

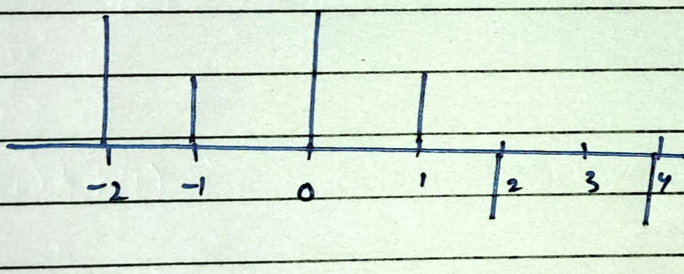
$\Rightarrow 2\left(x[n+1] + x[n-1]\right)$

$$x[n+1] = \delta[n+1] + 2[n+2] - \delta[n-2]$$

$$x[n+1] = \begin{cases} 1, & \cancel{n \in \{-1,-2,2\}}\ n=-1 \\ 2, & n=-2 \\ -1, & n=2 \end{cases}$$

$$x[n-1] = \delta[n-1] + 2\delta[n] - \delta[n-4]$$

$$x[n-1] = \begin{cases} 1, & n=1 \\ 2, & n=0 \\ -1, & n=4 \end{cases}$$



(4) $$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau-2)\, d\tau.$$